Lezione 3

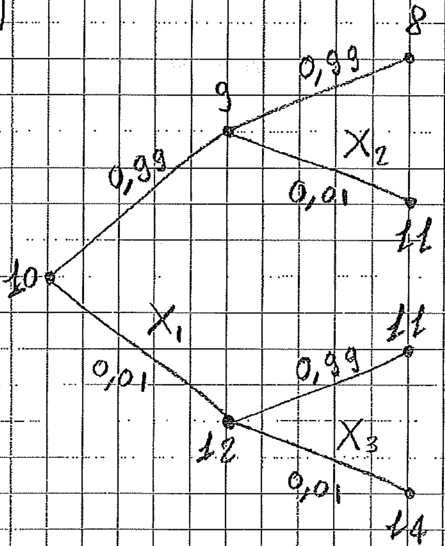
UNIVERSITÀ DEGLI STUDI DI VERONA

Laboratorio di R

Marco Minozzo

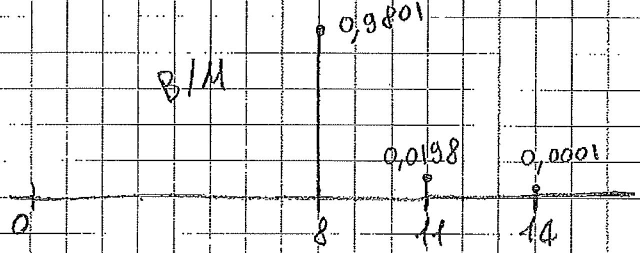
Esercizio 7 (Bonus Malus)

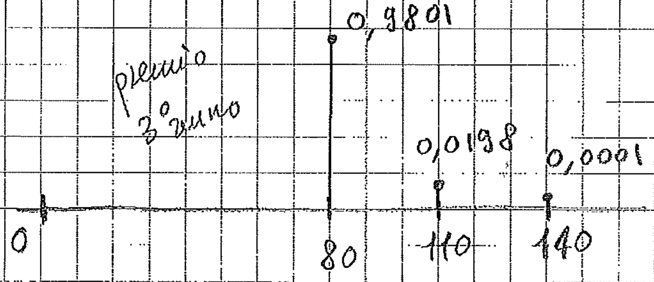
a)



ω	$P(\omega)$	B/M	premio 3° anno	premi	rimborsi
$\bar{s}\,\bar{s}$	0,9801	8	80	190	0
$\bar{s}\,s$	0,0099	11	110	190	X_2
$s\,\bar{s}$	0,0099	11	110	220	X_1
$s\,s$	0,0001	14	140	220	X_1+X_3
	1				

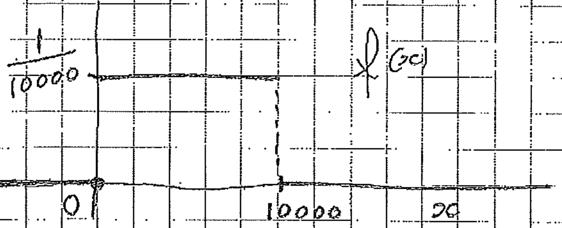
b)





c)

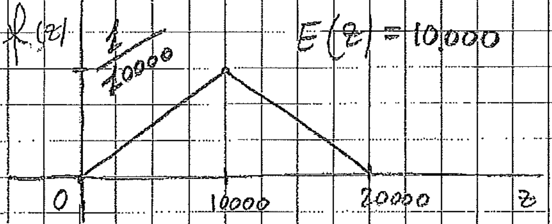
$X_i \sim U(0, 10000)$ $\quad i=1,2,3$ (indipendenti)



$E(X_i)=5000$, $\quad i=1,2,3$

Determiniamo la distribuzione di X_1+X_3, ovvero la distribuzione dei rimborsi se si verifica il quarto evento elementare "ss".

La somma $Z = X_1 + X_3$ ha densità "triangolare":



$E(Z) = 10.000$

$$f(z) = \begin{cases} \dfrac{z}{100\,000\,000}, & 0 < z < 10000, \\[2mm] \dfrac{2}{10000} - \dfrac{z}{100\,000\,000}, & 10000 < z < 20000, \\[2mm] 0, & \text{altrimenti} \end{cases}$$

Quindi,

$$P(\text{perdita}) = P(\text{perdita} \mid \bar{s}\bar{s}) \cdot P(\bar{s}\bar{s}) + P(\text{perdita} \mid \bar{s}s) \cdot P(\bar{s}s) + P(\text{perdita} \mid s\bar{s}) \cdot P(s\bar{s}) + P(\text{perdita} \mid ss) \cdot P(ss)$$

$$= 0 + P(X_2 > 190) \cdot P(\bar{s}s) + P(X_1 > 220) \cdot P(s\bar{s}) + P(X_1 + X_3 > 220) \cdot P(ss)$$

$$= 0 + (10000 - 190) \cdot \frac{1}{10000} \cdot 0{,}0099 + (10000 - 220) \cdot \frac{1}{10000} \cdot 0{,}0099 + \left[\int_{220}^{20000} f(z)\, dz \right] \cdot 0{,}0001$$

$$\int_{220}^{20000} f(z)\, dz = \int_{220}^{10000} \frac{z}{100000000}\, dz + \frac{1}{2} = \left[\frac{z^2}{200000000} \right]_{220}^{10000} + \frac{1}{2} = (0{,}5 - 0{,}000242) + \frac{1}{2} = 0{,}999758$$

$$= 0{,}981 \cdot 0{,}0099 + 0{,}978 \cdot 0{,}0099 + 0{,}999758 \cdot 0{,}0001$$

$$= 0{,}01949$$

d) $E(\text{premi} - \text{rimborsi}) = E(\text{premi}) - E(\text{rimborsi})$

$$E(\text{premi}) = 190 \cdot P(\bar{s}\bar{s}) + 190 \cdot P(\bar{s}s) + 220 \cdot P(s\bar{s}) + 220 \cdot P(ss)$$

$$= 190 \cdot 0{,}9801 + 190 \cdot 0{,}0099 + 220 \cdot 0{,}0099 + 220 \cdot 0{,}0001 = 190{,}3$$

$$E(\text{rimborsi}) = E(\text{rimborsi} \mid \bar{s}\bar{s}) \cdot P(\bar{s}\bar{s}) + E(\text{rimborsi} \mid \bar{s}s) \cdot P(\bar{s}s) + E(\text{rimborsi} \mid s\bar{s}) \cdot P(s\bar{s}) + E(\text{rimborsi} \mid ss) \cdot P(ss)$$

$$= 0 \cdot 0{,}9801 + 5000 \cdot 0{,}0099 + 5000 \cdot 0{,}0099 + 10000 \cdot 0{,}0001$$

$$= 100$$

$$E(\text{premi} - \text{rimborsi}) = E(\text{premi}) - E(\text{rimborsi}) = 190{,}3 - 100 = 90{,}3$$

Esercizio 8 (incasso giornaliero)

a) $X_1, X_2, \dots \sim \text{Esponenziale}(\lambda)$ $\quad E(X_i) = \frac{1}{\lambda} \quad Var(X_i) = \frac{1}{\lambda^2}$

$N \sim \text{Geometrica}(p)$ $\quad E(N) = \frac{1}{p} \quad Var(N) = \frac{1-p}{p^2}$

$$E(Y) = E\left(\sum_{i=1}^{N} X_i\right) = E\left[E\left(\sum_{i=1}^{N} X_i \,\Big|\, N\right)\right]$$

$$= E\Big[\underbrace{\frac{1}{\lambda} + \frac{1}{\lambda} + \dots + \frac{1}{\lambda}}_{N \text{ volte}}\Big] = E\left[N \cdot \frac{1}{\lambda}\right] = \frac{1}{\lambda} E(N) = \frac{1}{\lambda} \cdot \frac{1}{p}$$

$$Var(Y) = Var\left(\sum_{i=1}^{N} X_i\right) = Var\left[E\left(\sum_{i=1}^{N} X_i \,\Big|\, N\right)\right] + E\left[Var\left(\sum_{i=1}^{N} X_i \,\Big|\, N\right)\right]$$

$$= Var\left[N \cdot \frac{1}{\lambda}\right] + E\Big[\underbrace{\frac{1}{\lambda^2} + \frac{1}{\lambda^2} + \dots + \frac{1}{\lambda^2}}_{N \text{ volte}}\Big] = \frac{1}{\lambda^2} \cdot Var(N) + E\left[N \cdot \frac{1}{\lambda^2}\right]$$

$$= \frac{1}{\lambda^2} \cdot \frac{1-p}{p^2} + \frac{1}{\lambda^2} \cdot E(N) = \frac{1}{\lambda^2} \cdot \frac{1-p}{p^2} + \frac{1}{\lambda^2} \cdot \frac{1}{p} = \frac{1 \cancel{-p} \cancel{+p}}{\lambda^2 p^2} = \frac{1}{\lambda^2 p^2}$$

b) Per $y \geq 0$, la funzione di ripartizione di Y è data da:

$$F_Y(y) = P(Y \leq y) = \sum_{n=1}^{\infty} P(Y \leq y \mid N = n) \cdot P(N = n)$$

$$= \sum_{n=1}^{\infty} \left[\int_0^y \frac{1}{\Gamma(n)} \cdot \lambda^n u^{n-1} e^{-\lambda u} \, du\right] p (1-p)^{n-1}$$

$$= \int_0^y \lambda p e^{-\lambda u} \left(\sum_{n=1}^{\infty} \frac{[\lambda u (1-p)]^{n-1}}{(n-1)!}\right) du = \int_0^y \lambda p e^{-\lambda u} e^{\lambda u (1-p)} \, du$$

$$= \lambda p \int_0^y e^{-\lambda p u} \, du = 1 - e^{-\lambda p y},$$

mentre, per $y < 0$, $F_Y(y) = 0$. Ovvero, Y ha distribuzione esponenziale di parametro λp.

Lezione 3

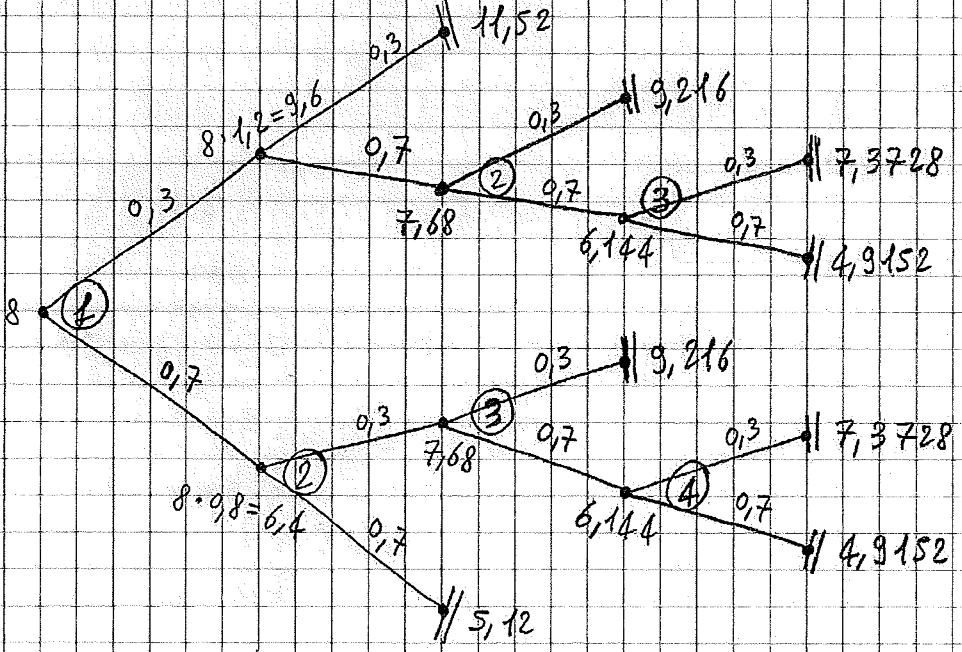
UNIVERSITÀ DEGLI STUDI DI VERONA

Laboratorio di R

Marco Minozzo

Esercizio 9 (strategia d'investimento)

a)



ω	$p(\omega)$	$X(\omega)$	$Y(\omega)$	perdita	Costi
++	0,3·0,3 = 0,09	11,52	1·11,52 = 11,52		8 = 8
+ − +	0,3·0,7·0,3 = 0,063	9,216	2·9,216 = 18,432		8 + 7,68 = 15,68
+ − − +	0,3·0,7·0,7·0,3 = 0,0441	7,3728	3·7,3728 = 22,1184		8 + 7,68 + 6,144 = 21,824
+ − − −	0,1029	4,9152	3·4,9152 = 14,7456	<	8 + 7,68 + 6,144 = 21,824
− + +	0,063	9,216	3·9,216 = 27,648		8 + 6,4 + 7,68 = 22,08
− + − +	0,0441	7,3728	4·7,3728 = 29,4912		8 + 6,4 + 7,68 + 6,144 = 28,224
− + − −	0,1029	4,9152	4·4,9152 = 19,6608	<	8 + 6,4 + 7,68 + 6,144 = 28,224
− −	0,49	5,12	2·5,12 = 10,24	<	8 + 6,4 = 14,4
	1				

b) $E(X^2) = (11{,}52)^2 \cdot 0{,}09 + (9{,}216)^2 \cdot 0{,}063 + (7{,}3728)^2 \cdot 0{,}0441 + (4{,}9152)^2 \cdot 0{,}1029 +$
$+ (9{,}216)^2 \cdot 0{,}063 + (7{,}3728)^2 \cdot 0{,}0441 + (4{,}9152)^2 \cdot 0{,}1029 + (5{,}12)^2 \cdot 0{,}49$
$= 45{,}257$

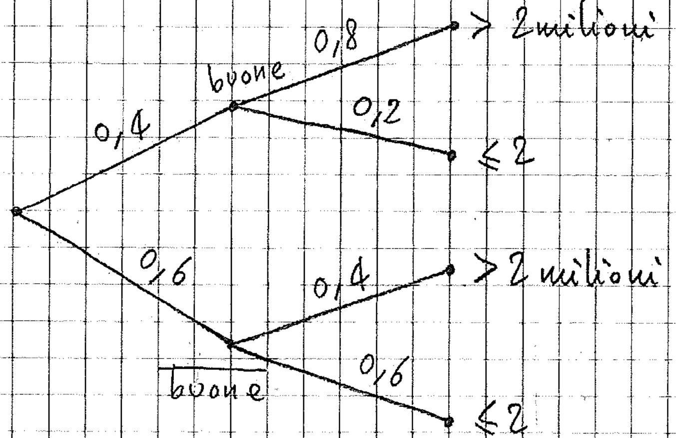
$E(Y) = 11{,}52 \cdot 0{,}09 + 18{,}432 \cdot 0{,}063 + 22{,}1184 \cdot 0{,}0441 + 14{,}7456 \cdot 0{,}1029 +$
$+ 27{,}648 \cdot 0{,}063 + 29{,}4912 \cdot 0{,}0441 + 19{,}6608 \cdot 0{,}1029 + 10{,}24 \cdot 0{,}49 = 14{,}7738$

c) $P(\text{perdita}) = P(+---) + P(-+--) + P(--)$
$= 0{,}1029 + 0{,}1029 + 0{,}49 = 0{,}6958$

d) $Y_\$ = 1{,}35 \cdot Y$
$E(Y_\$) = E(1{,}35 \cdot Y) = 1{,}35 \cdot E(Y) = 1{,}35 \cdot 14{,}7738 = 19{,}94463$

Esercizio 1 (occhiali da sole)

a)



ω	$p(\omega)$
$b>$	$0{,}4 \cdot 0{,}8 = 0{,}32$
$b\leq$	$0{,}4 \cdot 0{,}2 = 0{,}08$
$\bar{b}>$	$0{,}6 \cdot 0{,}4 = 0{,}24$
$\bar{b}\leq$	$0{,}6 \cdot 0{,}6 = 0{,}36$
	1

b) $P(>2) = 0{,}4 \cdot 0{,}8 + 0{,}6 \cdot 0{,}4 = 0{,}32 + 0{,}24 = 0{,}56$

c) $P(\text{buone} \mid >2) = \dfrac{P(\text{buone} \wedge >2)}{P(>2)}$
$= \dfrac{P(>2 \mid \text{buone}) \cdot P(\text{buone})}{P(>2)} = \dfrac{0{,}8 \cdot 0{,}4}{0{,}56} = 0{,}5714$